

QED Radiative Corrections to Asymmetries of Elastic ep-scattering in Hadronic Variables

A.V. Afanasev^{a)}, I. Akushevich^{a)*}, A.Ilyichev^{b)}, N.P.Merenkov^{c)}

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^(a) *North Carolina Central University, Durham, NC 27707, USA*
and

Jefferson Lab, Newport News, VA 23606, USA

^(b) *National Center of Particle and High Energy Physics, 220040 Minsk, Belarus*

^(c) *NSC "Kharkov Institute of Physics and Technology"*
63108, Akademicheskaya 1, Kharkov, Ukraine

Abstract

Compact analytical formulae for QED radiative corrections in the processes of elastic $e-p$ scattering are obtained in the case when kinematic variables are reconstructed from the recoil proton momentum measured. Numerical analysis is presented under kinematic conditions of current experiments at JLab.

1 Introduction

With the advent of new-generation electron accelerators such as CEBAF, experiments on elastic electron-proton scattering in the GeV-range were brought to the new level of precision. One of the novel features of the experiments is that in addition to the scattered electron, the momentum of recoil proton can be directly measured, while both the electron beam and the proton target may be polarized. The cross section depends only on one kinematic variable, so the measurement of final momenta gives several possibilities to reconstruct this kinematic variable from measurements. It results in different methods of data analysis, the best of them being chosen from experimental conditions such as resolution and others. Often it is used for reducing influence of radiative effects, which accompany any process with electrons. Ref.[1] presents a review of the radiative corrections calculations under different approaches of kinematic variable reconstructions in experiments at HERA.

In this paper we consider the case of elastic measurement

$$e(k_1) + p(p_1) \rightarrow e(k_2) + p(p_2) \quad (1)$$

where the initial electron is polarized longitudinally and only momentum of the final proton is measured or is used in $Q^2 = -(p_2 - p_1)^2$ reconstruction. The corresponding method is known as reconstruction within hadronic variables [1, 2]. There are two types of such experiments. In the first type the proton is considered to be polarized either longitudinally or transversely. In the second case the polarization states of recoil proton are measured. We calculate radiative corrections (see Fig.1) for these measurements and for hadronic variables reconstruction method.

Measurement of final proton momentum allows to define three independent kinematic variables. A natural choice for them are Q^2 , azimuthal angle and so-called inelasticity $u = (k_1 + p_1 - p_2)^2 - m^2$, where m is the electron mass. Since at the Born level $u = 0$ and $u > 0$ if the additional photon is emitted in the final state of (1), one can constrain (or cut) the range of u to reduce value of radiative corrections (RC).

We calculate RC to the cross sections and asymmetries differential in Q^2 . When leptonic variables reconstruction method is used, the integration over photonic momentum phase space cannot be performed analytically. The reason is that an argument of formfactors (Q^2 transferred to the proton) depends on the photon momentum. So the integration is left for numerical analysis in order to avoid additional assumptions about specific models for formfactors. The advantage of the reconstruction method in terms of hadronic variables is that the integration can be performed analytically and completely both for unpolarized and polarized parts of the cross

*on leave of absence from the National Center of Particle and High Energy Physics, 220040 Minsk, Belarus

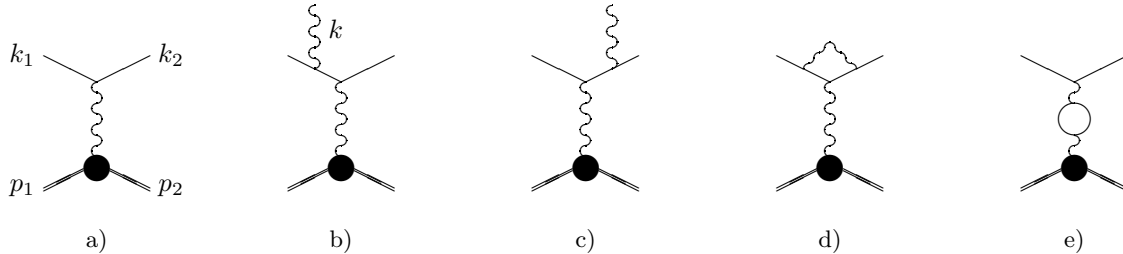


Figure 1: *Feynman diagrams contributing to the Born and the radiative correction cross sections.*

section. A more detailed discussion of advantages of RC calculation for elastic scattering within leptonic and hadronic variables can be found in ref.[3]

There are two possibilities in considering the polarization parts of the cross section. We consider different polarization states. It corresponds to different coefficients of expansion of polarization vectors, while the form of the expansion is the same. Since these coefficients depend on inelasticity (see Section 2), which is a function of photon momentum, we can produce either formulae for all specific polarization cases or get only one set of formulae for the polarization states but leaving integration over inelasticity for numerical analysis. We choose the second way. We will see in Section 3, that obtained formulae are very simple even in the general case.

We use the approach of Bardin and Shumeiko [4] for this calculation. Advantages of the approach are absence of any approximations both in the procedure of extraction and cancellation of infrared divergent terms and in integration over photon phase space. However it is known that mass singularity terms still remain within these formulae. In our case we perform additional procedure to extract these mass singularity terms. The result of the expansion can be presented as

$$\sigma = \log \frac{Q^2}{m^2} A(Q^2) + B(Q^2) + O\left(\frac{m^2}{Q^2}\right) \quad (2)$$

where independent on electron mass functions $A(Q^2)$ and $B(Q^2)$ appear as LO and NLO contribution to the cross section. The explicit formulae in the form of (2) are presented in Section 3.

Section 4 is devoted to numerical analysis within kinematic conditions of experiments at JLab. We analyze RC to unpolarized cross section and different asymmetries accessed in polarization measurements. Furthermore, obtained results can be applied for other reactions, where $ep \rightarrow ep\gamma$ is a background process. In Section 4 a specific example is considered when the process is background to measurements of pion production by polarized real photons at JLab.

2 The Born cross section and kinematics

The Born cross section of the process (1) can be written in the form

$$d\sigma_0 = \frac{M_0^2}{4pk_1} d\Gamma_0 = M_0^2 \frac{dQ^2}{16\pi S^2} \quad (3)$$

where $S = 2k_1p$. Kinematic limits for Q^2 are defined as

$$0 \leq Q^2 \leq \frac{\lambda_s}{S + m^2 + M^2}, \quad \lambda_s = S^2 - 4m^2M^2 \quad (4)$$

where (m, M) are the electron and proton masses). Here, for generality, we keep the electron mass, however, below we will neglect it wherever possible. At the Born level the mass can be set to zero in the final formulae, while for RC cross section we will construct expansion (2). We note that in general we should keep explicit azimuthal angular dependence. The cross section (both Born and RC ones) does not depend on this angle, but kinematic cuts can. We assumed that there is only a cut on inelasticity or all other cuts can be effectively reduced to it. In this case integration over the azimuthal angle can be performed analytically, that results to (3). Born matrix element is the contraction of leptonic and hadronic tensors:

$$M_0^2 = \frac{e^4}{Q^4} L_{\mu\nu}^0 W_{\mu\nu} \quad (5)$$

We use standard definitions for the leptonic tensor,

$$L_{\mu\nu}^0 = \frac{1}{2} \text{Tr}(\hat{k}_2 + m)\gamma_\mu(1 + \gamma_5 \hat{\xi})(\hat{k}_1 + m)\gamma_\nu. \quad (6)$$

The hadronic tensor can be described by four structure functions in all considered cases:

$$W_{\mu\nu} = \sum_i w_{\mu\nu}^i \mathcal{F}_i \quad (7)$$

with

$$w_{\mu\nu}^1 = -g_{\mu\nu} \quad w_{\mu\nu}^2 = \frac{p_\mu p_\nu}{M^2} \quad w_{\mu\nu}^3 = -i\epsilon_{\mu\nu\lambda\sigma} \frac{q_\lambda \eta_\sigma}{M}, \quad w_{\mu\nu}^4 = i\epsilon_{\mu\nu\lambda\sigma} \frac{q_\lambda p_\sigma \eta q}{M^3} \quad (8)$$

and ($\tau_p = Q^2/4M^2$)

$$\mathcal{F}_1 = 4\tau_p M^2 G_m^2 \quad \mathcal{F}_2 = 4M^2 \frac{G_E^2 + \tau_p G_m^2}{1 + \tau_p} \quad \mathcal{F}_3 = -2M^2 G_E G_M, \quad \mathcal{F}_4 = -M^2 G_M \frac{G_E - G_M}{1 + \tau_p}. \quad (9)$$

Polarization effects are described by polarization four-vectors of the lepton (ξ) and proton (η). Following [5] we expand them over the measured momenta k_1 , p_1 and p_2

$$\begin{aligned} \xi &= \frac{2}{\sqrt{\lambda_s}} \left(\frac{S}{2m} k_1 - m p_1 \right) \\ \eta &= 2a_\eta k_1 + b_\eta q + c_\eta (p_1 + p_2) \end{aligned} \quad (10)$$

where $q = p_1 - p_2$. In this paper we consider four¹ polarization states, when initial or final protons are polarized along or perpendicular (in the scattering plane) to the vector \vec{q} . For these cases the following expressions of the coefficients are used,

$$a_\eta^L = 0, \quad b_\eta^L = \frac{Q^2 + 4M^2}{2M\sqrt{\lambda_M}}, \quad c_\eta^L = \frac{Q^2}{2M\sqrt{\lambda_M}} \quad (11)$$

and

$$a_\eta^T = \frac{Q^2(Q^2 + 4M^2)}{2\sqrt{\lambda_h}\sqrt{\lambda_M}}, \quad b_\eta^T = \frac{(Q^2 + 4M^2)Q_u^2}{2\sqrt{\lambda_h}\sqrt{\lambda_M}}, \quad c_\eta^T = -\frac{Q^2(2S - Q_u^2)}{2\sqrt{\lambda_h}\sqrt{\lambda_M}}. \quad (12)$$

Here $Q_u^2 = Q^2 + u$, $\lambda_h = SQ^2(S - Q_u^2) - M^2Q_u^4 - m^2\lambda_M$ and $\lambda_M = Q^2(Q^2 + 4M^2)$. These formulae are given for the case of initial particles polarizations. For the final polarizations these formulae can be kept but $b_\eta^L \rightarrow -b_\eta^L$. One can make sure that initial and final polarization vectors exactly satisfy the necessary conditions:

$$\eta_{L,T}^2 = -1, \quad \eta_L \eta_T = 0, \quad q\eta_T = 0, \quad \bar{p}\eta_{L,T} = 0 \quad (13)$$

where \bar{p} is p_1 or p_2 in dependence of initial or final polarization vector is considered.

Calculating the contractions in (5) we obtain the Born cross section in the form

$$\frac{d\sigma_0}{dQ^2} = \frac{2\pi\alpha^2}{S^2Q^4} \sum_i \theta_B^i \mathcal{F}_i, \quad (14)$$

where

$$\begin{aligned} \theta_1^0 &= 2Q^2 \\ \theta_2^0 &= \frac{1}{M^2}(S^2 - Q^2S - M^2Q^2) \\ \theta_3^0 &= -\frac{2Q^2}{M}(Q^2a_\eta + (2S - Q^2)c_\eta) \\ \theta_4^0 &= \frac{Q^4}{M^3}(2S - Q^2)(a_\eta - 2b_\eta) \end{aligned}$$

¹The considered model independent RC does not include box-type contributions and therefore cannot lead to the additional T-odd polarization and/or asymmetry. The polarization part of the cross section of the model independent RC as well as of the Born cross section is exactly zero for normal polarization

3 Radiative corrections

The cross section of radiative process (Fig. 1b,c)

$$e(k_1) + p(p_1) \longrightarrow e'(k_2) + \gamma(k) + p(p_2), \quad (15)$$

can be presented in a general form

$$d\sigma_r = \frac{M_r^2}{4pk_1} d\Gamma_r \quad (16)$$

The phase space of three final particles

$$d\Gamma_r = \frac{1}{(2\pi)^5} \frac{d^3p_2}{2p_{20}} \frac{d^3k_2}{2k_{20}} \frac{d^3k}{2k_0} \delta(p + k_1 - k_2 - k - p_2) \quad (17)$$

can be parameterized in terms of four invariant variables [6, 7]: Q^2 , $u = 2k_2k$, $w = 2k_1k$ and $z_2 = 2p_2k$

$$d\Gamma_r = \frac{dQ^2}{8(2\pi)^4 S} \int_0^{u_m} du \int_{w_{min}}^{w_{max}} dw \int_{z_{min}}^{z_{max}} \frac{dz_2}{\sqrt{R_z}} \quad (18)$$

where R_z comes from Gram determinant and coincides with a standard R_z -function from Bardin-Shumeiko approach [4, 6]. The formalism to calculate it is general and is developed in [8]. Explicit formulae for analytical integration are presented in papers [6, 7, 3]. Note that integration over the azimuthal angle is assumed to be performed (see discussion after Eq.(4)).

The matrix element squared of radiated process reads

$$M_r^2 = \frac{e^6}{Q_h^4} L_{\mu\nu}^r W_{\mu\nu} \quad (19)$$

The leptonic tensors of radiative process in (19) looks like

$$L_{\mu\nu}^r = -\frac{1}{2} Tr (\hat{k}_2 + m) \Gamma_{\mu\alpha} (1 + \gamma_5 \hat{\xi}) (\hat{k}_1 + m) \bar{\Gamma}_{\alpha\nu}, \quad (20)$$

where

$$\Gamma_{\mu\alpha} = \left[\left(\frac{k_{1\alpha}}{kk_1} - \frac{k_{2\alpha}}{kk_2} \right) \gamma_\mu - \frac{\gamma_\mu \hat{k} \gamma_\alpha}{2kk_1} - \frac{\gamma_\alpha \hat{k} \gamma_\mu}{2kk_2} \right], \bar{\Gamma}_{\alpha\nu} = \left[\left(\frac{k_{1\alpha}}{kk_1} - \frac{k_{2\alpha}}{kk_2} \right) \gamma_\nu - \frac{\gamma_\alpha \hat{k} \gamma_\nu}{2kk_1} - \frac{\gamma_\nu \hat{k} \gamma_\alpha}{2kk_2} \right]. \quad (21)$$

The calculation of the cross section of the radiative process is not so straightforward because of infrared divergence. The separation of the infrared divergence can be performed in a standard way [4]:

$$d\sigma_r = d\sigma_r^{IR} + d\sigma_r - d\sigma_r^{IR} = d\sigma_r^{IR} + d\sigma_r^F. \quad (22)$$

The first term in r.h.s. of Eq. (22) has an infrared divergence and can be presented in the form:

$$d\sigma_r^{IR} = \frac{\alpha}{\pi} \delta^{IR} d\sigma_0 = \frac{\alpha}{\pi} (\delta_S + \delta_H) d\sigma_0 \quad (23)$$

while the second one in (22) is finite for $k \rightarrow 0$. The quantities δ_S and δ_H appear after additional splitting the integration region over inelasticity u by the infinitesimal parameter \bar{u}

$$\begin{aligned} \delta_S &= \frac{-1}{\pi} \int_0^{\bar{u}} du \int \frac{d^{n-1}k}{(2\pi\mu)^{n-4}k_0} F_{IR} \delta((\Lambda_h - k)^2 - m^2) \\ \delta_H &= \frac{-1}{\pi} \int_{\bar{u}}^{u_m} du \int \frac{d^3k}{k_0} F_{IR} \delta((\Lambda_h - k)^2 - m^2) \end{aligned} \quad (24)$$

where $\Lambda_h = k_1 + p_1 - p_2$ and

$$F_{IR} = \frac{Q^2}{uw} - \frac{m^2}{u^2} - \frac{m^2}{w^2}. \quad (25)$$

For this specific procedure it is convenient to keep integration over photon momentum as it is. In this case one delta function appears in the integrand. It is equivalent to integration over w and z_2 in (18). The integration gives the following results

$$\begin{aligned}\delta_S &= 2\left(P^{IR} + \log \frac{\bar{u}}{m\mu}\right)(l_m - 1) + 1 + l_m - l_m^2 - \frac{\pi^2}{6}, \\ \delta_H &= 2(l_m - 1) \log \frac{u_m}{\bar{u}} - \frac{1}{2}(l_v + l_m)^2 + l_v + l_m - l_w(l_m + l_w - l_v) - \frac{\pi^2}{6} - \text{Li}_2\left(-\frac{u_m}{Q^2}\right), \\ l_m &= \log \frac{Q^2}{m^2}, \quad l_v = \log \frac{u_m}{Q^2}, \quad l_w = -\log(x_m), \quad x_m = \frac{Q^2}{Q^2 + u_m}.\end{aligned}\quad (26)$$

The explicit expression for the additional photon exchange contribution coming from diagram (1d) has also the factorized form:

$$d\sigma_V = \frac{\alpha}{\pi} \delta^V d\sigma_0 \quad (27)$$

where

$$\delta^V = -2\left(P^{IR} + \log \frac{m}{\mu}\right)(l_m - 1) - \frac{1}{2}l_m^2 + \frac{3}{2}l_m - 2 + \frac{\pi^2}{6} \quad (28)$$

Thus the sum of contributions (23) and (27)

$$\delta^{el} = \delta^V + \delta_S + \delta_H^{IR} = l_m(l_v - l_w + \frac{3}{2}) - l_v - 1 - \frac{3}{2}l_w^2 - \frac{1}{2}l_v^2 + 2l_w l_v - \text{Li}_2(x_m) \quad (29)$$

is infrared free.

The second term in (22) is infrared free. Hence this contribution can be written using the expression for photon phase space (18) as

$$d\sigma^F = \frac{\alpha^3}{4} \frac{dQ^2}{S^2 Q^4} \sum_{i=1}^4 \theta_i^F \mathcal{F}_i. \quad (30)$$

where only coefficients θ_i^F include integrals:

$$\theta_i^F = \frac{1}{\pi} \int_0^{u_m} du \int_{w_{min}}^{w_{max}} dw \int_{z_{min}}^{z_{max}} \frac{dz_2}{\sqrt{R_z}} [L_{\mu\nu}^r - \alpha F_{IR} L_{\mu\nu}^0] w_{\mu\nu}^i = \frac{1}{\pi} \int_{x_m}^1 \frac{Q^2}{x^2} dx \int_{w_{min}}^{w_{max}} dw \int_{z_{min}}^{z_{max}} \frac{dz_2}{\sqrt{R_z}} [L_{\mu\nu}^r - \alpha F_{IR} L_{\mu\nu}^0] w_{\mu\nu}^i. \quad (31)$$

Here we use the transformation of external variable u ,

$$x = \frac{Q^2}{u + Q^2}, \quad (32)$$

to have final results in the simplest form, where x_m is defined in Eq.(26).

Integrals in (31) can be calculated analytically completely. However, as we discussed in Introduction, one integration in polarization part of the cross section is retained for generality:

$$\begin{aligned}\theta_1^F &= 2Q^2 \left[\frac{1}{x_m} (2L - 1) + L + 2l_m(l_w - 3) - 2l_w + l_w^2 - 2\text{Li}_2(x_m) + \frac{\pi^2}{3} \right] \\ \theta_2^F &= \frac{1}{M^2} \left\{ Q^4 l_w - Q^2 (S + M^2) [L + 2l_m(l_w - 3) + l_w^2 - 2\text{Li}_2(x_m) + \frac{\pi^2}{3}] + \frac{M^2 Q^2}{x_m} [1 - 2L] \right. \\ &\quad \left. + S(Q^2 - S) [2x_m(1 - L) - 4l_w - 1] + S^2 [x_m^2 L + 3 - x_m - 2x_m^2 - 6l_m] \right\} \\ \theta_3^F &= \frac{2Q^2}{M} \int_{x_m}^1 \frac{dx}{x} \left[Q^2 a_\eta h_1 + (2Sh_2 + Q^2 h_3) c_\eta + (Q^2 \Delta a_\eta + (2S - Q^2) \Delta c_\eta) h_4 \right] + \theta_3^0 \delta_{pol}^F \\ \theta_4^F &= \frac{Q^4}{M^3} \int_{x_m}^1 \frac{dx}{x} \left[(Q^2 h_1 + 2Sh_3) a_\eta + (2Sh_2 + Q^2 h_3) b_\eta + (2S - Q^2) h_4 (\Delta b_\eta - \Delta a_\eta) \right] + \theta_4^0 \delta_{pol}^F\end{aligned}\quad (33)$$

Here the contribution of polarization part is presented in such a form where x (or u) dependence is included only in polarization coefficients and in functions:

$$\begin{aligned} h_1 &= (3x^2 - x + 5 - 2l_x(2x^2 + x + 1))/x^2, \\ h_2 &= 1 - 8x - 2l_x(1 - x), \\ h_3 &= (5x - 5 + 2l_x(x + 1))/x, \\ h_4 &= (3x + 4 - 4l_x x)/(1 - x). \end{aligned} \quad (34)$$

The special procedure of additional subtraction was applied for polarization contribution in order to extract mass singularity terms to l_m , which is contained in $l_x = l_m - \log(x(1 - x))$ and $L = l_m - l_v + 2l_w$ now. In subtracted (and added) terms arguments of polarization coefficients were taken for $x = 1$ (or $u = 0$) as for Born process. As a result differences $\Delta\{a, b, c\}_\eta = \{a, b, c\}_\eta(x) - \{a, b, c\}_\eta(1)$ appeared in the integrands. Because of added terms does not contain x -dependence of polarization coefficients, they can be integrated explicitly. This is the origin of factorized terms with δ_{pol}^F in expressions for θ_3^F and θ_4^F :

$$\delta_{pol}^F = - \int_{x_m}^1 dx \left[\frac{x^4 Q^2 (3Q^2(1 - x) + 4xm^2)}{(Q^2(1 - x) + m^2 x)^2} + 3x^3 + 3x^2 + 3x + 3 \right] \quad (35)$$

$$= -3(l_m + l_v - l_w) - 1 \quad (36)$$

The last contribution which has to be taken into account (see Fig. 1e) is vacuum polarization by leptons and by hadrons ($d\sigma_{vac}$). Leptonic contribution comes from QED (see [5], for example), while hadronic contribution can be obtained from experimental data for the process $e^+e^- \rightarrow hadrons$. We will use a model developed in ref.[9]. The vacuum polarization correction has also a factorized form and we denote them δ_{vac}^l and δ_{vac}^h for leptonic and hadronic contributions.

The final result for the RC cross section is obtained by adding all the considered contributions:

$$d\sigma_V + d\sigma_r + d\sigma_{vac} = \frac{\alpha^3}{4} \frac{dQ^2}{S^2 Q^4} \sum_{i=1}^4 [\theta_i^F + 4(\delta^{el} + \delta_{vac}^l + \delta_{vac}^h)\theta_i^0] \mathcal{F}_i. \quad (37)$$

This expression is the correction to the cross section differential in Q^2 only. Sometimes it is necessary to have formulae for the cross section versus inelasticity also. They can be obtained by straightforward differentiation over x_m (or u_m) of the result (37):

$$\frac{d\sigma_r}{dx} = \frac{d\sigma_r}{du} \frac{Q^2}{x^2} = \frac{\alpha^3}{4} \frac{dQ^2}{S^2 Q^4} \sum_{i=1}^4 \frac{d\theta_i^R}{dx} \mathcal{F}_i. \quad (38)$$

where

$$\begin{aligned} \frac{d\theta_1^R}{dx} &= -2Q^2 \frac{2(1+x^2)L + 1 - 8x}{(1-x)x^2} \\ \frac{d\theta_2^R}{dx} &= \frac{1}{M^2} \left\{ \frac{4(x^2 S(xS - Q^2) - M^2 Q^2)}{(1-x)x} - \frac{Q^4}{x} \right. \\ &\quad \left. - \frac{(xS(xS - Q^2) - M^2 Q^2)[2(1+x^2)L + 1 - 2x(1+x)]}{(1-x)x^2} \right\} \\ \frac{d\theta_3^R}{dx} &= \frac{2Q^2(2(1+x^2)L - 8x^2 + 6x - 5)(Q^2 a_\eta + (2xS - Q^2)xc_\eta)}{(1-x)x^3 M} \\ \frac{d\theta_4^R}{dx} &= \frac{Q^4(2(1+x^2)L - 8x^2 + 6x - 5)(2xS - Q^2)(xb_\eta - a_\eta)}{(1-x)x^3 M^3} \end{aligned} \quad (39)$$

It is clear that only hard photon radiation can contribute to the functions. The terms containing x_m in δ' s and in subtracted parts $d\sigma_r^{IR}$ have to be completely canceled. It can be seen in polarization parts that the terms containing $a_\eta(1)$, $b_\eta(1)$ and $c_\eta(1)$ explicitly cancel out in final expressions (39).

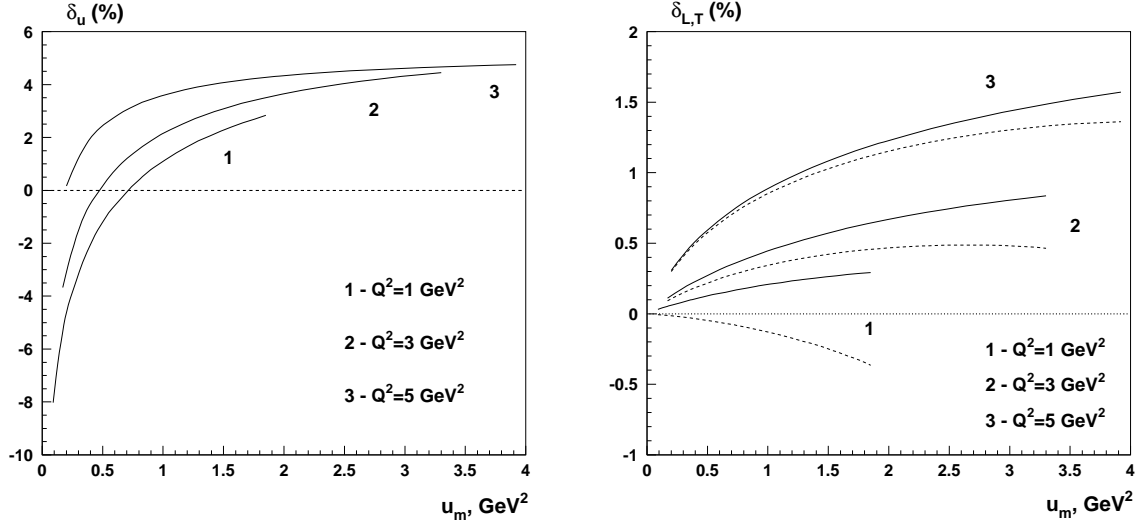


Figure 2: Radiative corrections to the unpolarized cross section (left plot) and polarization asymmetries (right plot) defined in (41). Solid and dashed lines corresponds to longitudinal and transverse cases. $S=8 \text{ GeV}^2$.

4 Numerical analysis

The obtained results can have direct application in experiments at JLab, so we perform the numerical analysis within kinematic conditions of JLab. First we consider the unpolarized cross section and RC to it. The next step is consideration of polarization effects in two cases when initial or final protons are polarized.

Both the Born and observed cross sections can be split into unpolarized and polarized parts:

$$\frac{d\sigma^{born,obs}}{dQ^2} = \sigma_u^{b,obs} \pm \sigma_p^{b,obs} \quad (40)$$

We consider four different polarization states described in Eqs.(11,12), so we have four different polarized parts of cross sections, which, of course, corresponds to only one unpolarized cross section. Let us define the relative corrections to the observable quantities in the current experiments:

$$\begin{aligned} \delta_u &= \frac{\sigma_u^{obs}}{\sigma_u^b} - 1, & \delta_{L,T} &= \left[\frac{\sigma_p^{obs}}{\sigma_u^{obs}} - \frac{\sigma_p^b}{\sigma_u^b} \right] \left[\frac{\sigma_p^b}{\sigma_u^b} \right]^{-1} = \frac{\sigma_u^b}{\sigma_u^{obs}} \frac{\sigma_p^{obs}}{\sigma_p^b} - 1 \\ \delta_r &= \left[\frac{\sigma_{pT}^{obs}}{\sigma_{pL}^{obs}} - \frac{\sigma_{pT}^b}{\sigma_{pL}^b} \right] \left[\frac{\sigma_{pT}^b}{\sigma_{pL}^b} \right]^{-1} = \frac{\sigma_{pL}^b}{\sigma_{pL}^{obs}} \frac{\sigma_{pT}^{obs}}{\sigma_{pT}^b} - 1 \end{aligned} \quad (41)$$

The first correction δ_u is the relative correction to unpolarized cross section. The $\delta_{L,T}$ are corrections to polarization asymmetries measured by rotating the polarization states of initial protons. At last, the quantity δ_r is the correction to the measured ratio of final proton polarizations [10, 11]. The correction to the unpolarized cross section is presented in Figure 2a. The behavior is quite typical. For the very hard inelasticity cut ($u_m \ll Q^2$) the positive contribution due to real bremsstrahlung is suppressed, so there is only negative loop correction contributing to cross section. Different ending values for the curves corresponds to different kinematically allowed regions.

The correction to the asymmetry is given in the plot 2b. The magnitude of the correction does not exceed 1.5%. It goes up with increasing of Q^2 and inelasticity cut. Second effect is clear because the only unfactorized (different for unpolarized and polarized parts) hard bremsstrahlung can contribute to RC to asymmetries. The Q^2 -dependence is also understood because of contribution of a large logarithm $\log(Q^2/m^2)$. Both the cross sections and asymmetries have non-zero leading contributions, so the correction is larger when Q^2 is going up.

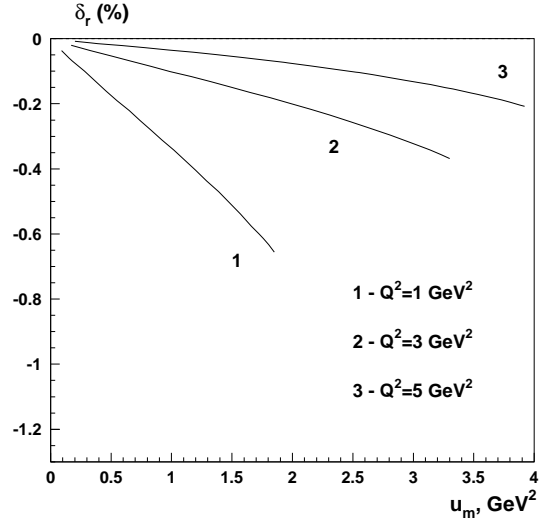


Figure 3: Radiative correction to recoil proton polarization for (43) in the region the invariant mass of the unobserved state close to the pion mass. $S=8\text{GeV}^2$.

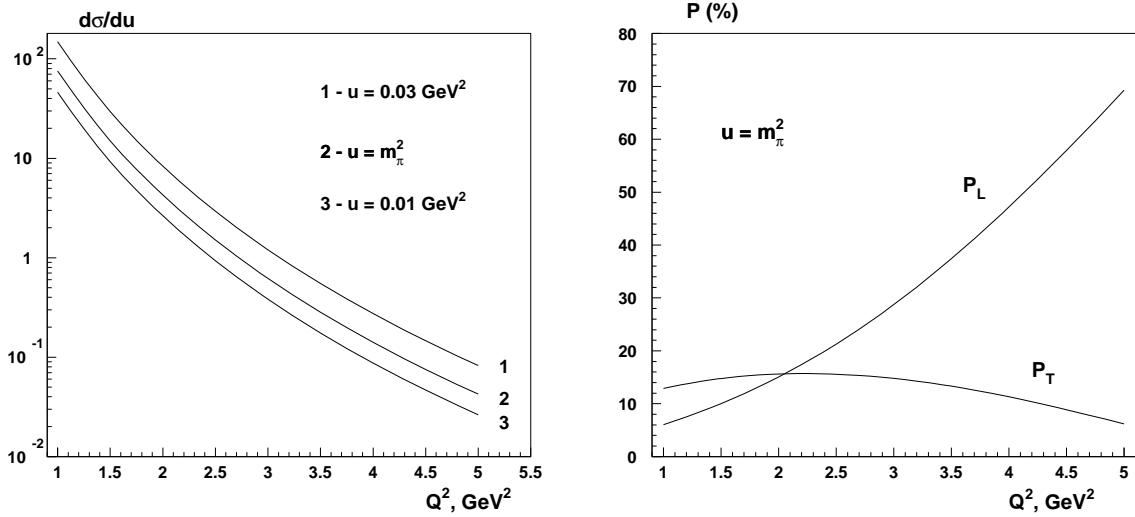


Figure 4: The cross section and proton polarizations of the process (43). Q^2 here is equivalent to $-t$ defined in [13].

In the case of calculation of correction to the transferred polarization experiment the correction is negative and does not exceed 1%. It is in agreement with our earlier estimation of the effect [12, 3].

An interesting application of our result can be found for the process

$$\vec{\gamma} + p \rightarrow \pi^0 + \vec{p} \quad (42)$$

recently analyzed in JLab experiment [13]. Usually the photons in the reaction are produced by polarized electron beam. As a result both photon and electrons are in the beam. Therefore when invariant mass of undetected e and γ is close to the pion mass m_π the process considered in this paper

$$\vec{e} + p \rightarrow e + \gamma + \vec{p} \quad (43)$$

can be background to the process (42). In the end of the section we analyze unpolarized cross section and recoil proton polarization due to the process (43).

In Figure 4a we show the cross section vs Q^2 for several values of the unobserved invariant masses set close to the pion mass. Plot 4b shows polarization of protons due to the bremsstrahlung process (43) being background to the measurements [13].

5 Conclusion

In this paper we consider the radiative effects in elastic electron-proton scattering with the hadronic variables reconstruction method. Within this method the information on proton final momentum is used to reconstruct the kinematic variable Q^2 . Another kinematic variable which can be also reconstructed from the measured recoil momentum of proton is inelasticity. The Born cross section does not depend on the inelasticity, so this fact may be used in measurements to make a cut on this variables. It allows to reduce radiative corrections essentially, but at the cost of significant loss in statistics. In this paper we obtain explicit formulae for the cross section and spin asymmetries versus Q^2 and inelasticity u . We calculated RC to unpolarized cross section and polarization observables in kinematic conditions of experiments held at JLab. We found that correction is about one per cent. It increases when both Q^2 and inelasticity go up. Also we calculated a cross section and proton polarization from elastic ep-events that produce background in neutral pion production by polarized real photon at JLab.

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